TRIGONOMÉTRIE

A. Cercle trigonométrique et valeurs remarquables

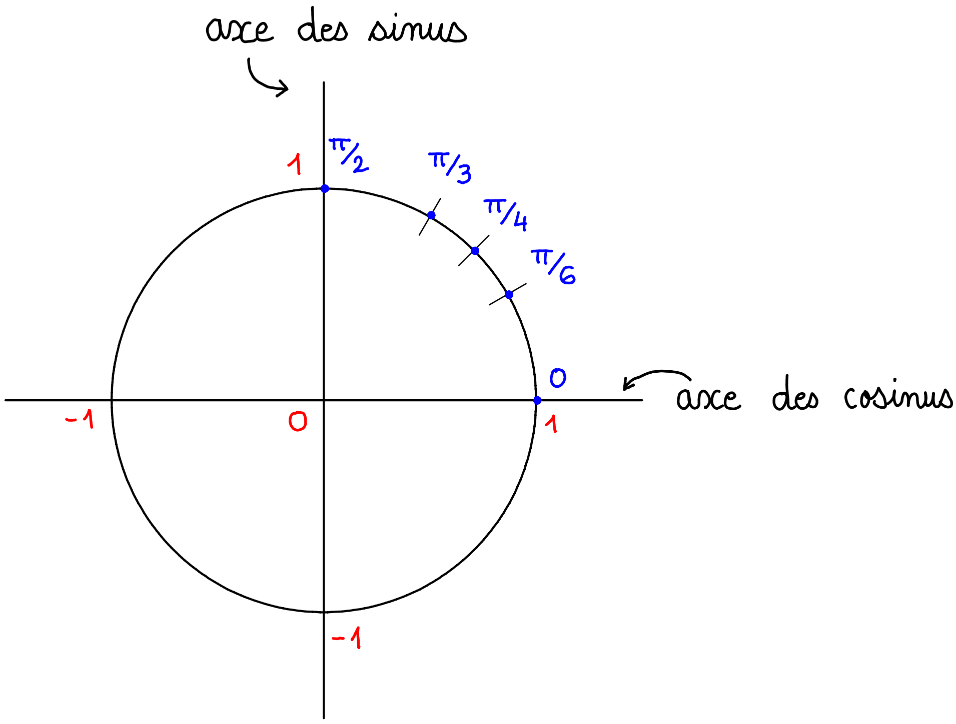


Tableau des valeurs remarquables

α en rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
α en °	0	30	45	60	90
sin	0	$\frac{1}{2} = 0,5$	$\frac{\sqrt{2}}{2} \approx 0,7$	$\frac{\sqrt{3}}{2} \approx 0,8$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3} \approx 0,6$	1	$\sqrt{3} \approx 1,7$	$\|\|$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

Formule de conversion

$$\frac{\alpha_1}{180^\circ} = \frac{\alpha_2}{\pi \text{ rad}}$$

Théorème de Pythagore

$$\sin^2 \alpha + \cos^2 \alpha = 1$$
$$(\sin \alpha)^2 + (\cos \alpha)^2 = 1$$

Domaine d'existence

$$-1 \leq \sin \alpha \leq +1$$
$$-1 \leq \cos \alpha \leq +1$$

010 À partir des informations précédentes compléter sur $[\ 0\ ;\ 2\pi\ ]$:

α , en radians	0	$\frac{\pi}{6}$	$\frac{2\pi}{6} =$	$\frac{3\pi}{6} =$	$\frac{4\pi}{6} =$	$\frac{5\pi}{6}$	$\frac{6\pi}{6} =$	$\frac{7\pi}{6}$	$\frac{8\pi}{6} =$	$\frac{9\pi}{6} =$	$\frac{10\pi}{6} =$	$\frac{11\pi}{6}$	$\frac{12\pi}{6} =$
α , en degrés													
sin													
cos													

α , en radians	0	$\frac{\pi}{4}$	$\frac{2\pi}{4} =$	$\frac{3\pi}{4}$	$\frac{4\pi}{4} =$	$\frac{5\pi}{4}$	$\frac{6\pi}{4} =$	$\frac{7\pi}{4}$	$\frac{8\pi}{4} =$
α , en degrés									
sin									
cos									

α , en radians	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{3\pi}{3} =$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{6\pi}{3} =$
α , en degrés							
sin							
cos							

α , en radians	0	$\frac{\pi}{2}$	$\frac{2\pi}{2} =$	$\frac{3\pi}{2}$	$\frac{4\pi}{2} =$
α , en degrés					
sin					
cos					

020 À partir des informations précédentes compléter sur $[\ -\pi\ ;\ \pi\ ]$:

α , en radians	$\frac{-6\pi}{6} =$	$\frac{-5\pi}{6}$	$\frac{-4\pi}{6} =$	$\frac{-3\pi}{6} =$	$\frac{-2\pi}{6} =$	$\frac{-\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{2\pi}{6} =$	$\frac{3\pi}{6} =$	$\frac{4\pi}{6} =$	$\frac{5\pi}{6}$	$\frac{6\pi}{6} =$
α , en degrés													
sin													
cos													

α , en radians	$\frac{-4\pi}{4} =$	$\frac{-3\pi}{4}$	$\frac{-2\pi}{4} =$	$\frac{-\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{2\pi}{4} =$	$\frac{3\pi}{4}$	$\frac{4\pi}{4} =$
α , en degrés									
sin									
cos									

α , en radians	$\frac{-3\pi}{3} =$	$\frac{-2\pi}{3}$	$\frac{-\pi}{3}$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{3\pi}{3} =$
α , en degrés							
sin							
cos							

α , en radians	$\frac{-2\pi}{2} =$	$\frac{-\pi}{2}$	0	$\frac{\pi}{2}$	$\frac{2\pi}{2} =$
α , en degrés					
sin					
cos					

B. Équations trigonométriques

- X est la variable qui peut s'écrire sous différentes formes : x, $2x$, $-x$, $\frac{x}{3}$, $x+\frac{\pi}{4}$...

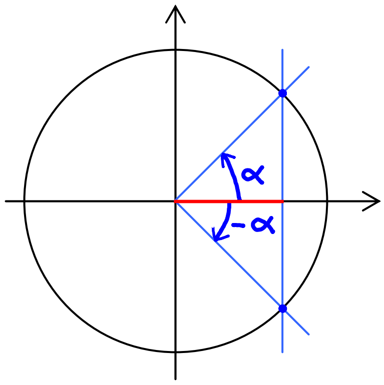
$$\cos X = a$$

or $\alpha = \cos^{-1} a = \arccos a$ donc

$$\cos X = \cos \alpha$$

Les solutions dans $\mathbb{R}$ sont :

$$\begin{cases} X = \alpha + k \times 2\pi \\ \text{ou} \\ X = -\alpha + k \times 2\pi \end{cases}, \quad k \in \mathbb{Z}$$



Exemple : Résoudre dans $\mathbb{R}$ $\cos x = \frac{1}{2}$

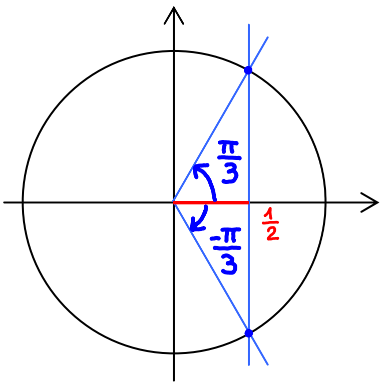
or $\alpha = \cos^{-1}\frac{1}{2} = \arccos\frac{1}{2} = \frac{\pi}{3}$ donc

$$\cos x = \cos\frac{\pi}{3}$$

Les solutions dans $\mathbb{R}$ sont :

$$\begin{cases} x = \frac{\pi}{3} + k \times 2\pi \\ \text{ou} \\ x = -\frac{\pi}{3} + k \times 2\pi \end{cases}, \quad k \in \mathbb{Z}$$

$$S_{\mathbb{R}} = \left\{ -\frac{\pi}{3} + k \times 2\pi \; ; \; \frac{\pi}{3} + k \times 2\pi \quad , \; k \in \mathbb{Z} \right\}$$



- X est la variable qui peut s'écrire sous différentes formes : x, $2x$, $-x$, $\frac{x}{3}$, $x+\frac{\pi}{4}$...

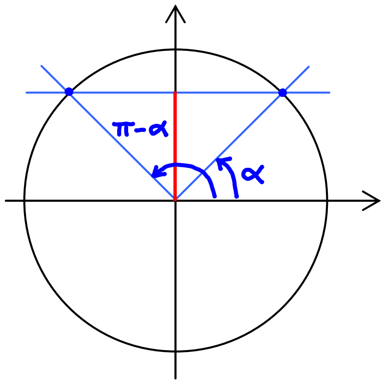
$$\sin X = b$$

or $\alpha = \sin^{-1} b = \arcsin b$ donc

$$\sin X = \sin \alpha$$

Les solutions dans ℝ sont :

$$\begin{cases} X = \alpha + k \times 2\pi \\ \text{ou} \\ X = (\pi - \alpha) + k \times 2\pi \end{cases}, \quad k \in \mathbb{Z}$$



Exemple : Résoudre dans ℝ

$$\sin x = \frac{\sqrt{3}}{2}$$

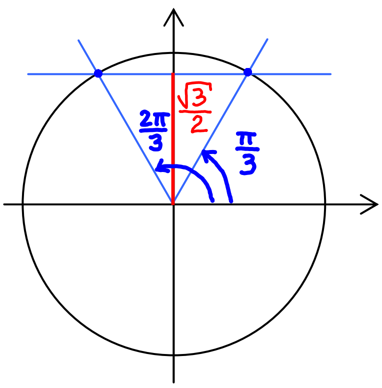
or $\alpha = \sin^{-1}\frac{\sqrt{3}}{2} = \arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ donc

$$\sin x = \sin\frac{\pi}{3}$$

Les solutions dans ℝ sont :

$$\begin{cases} x = \frac{\pi}{3} + k \times 2\pi \\ \text{ou} \\ x = \pi - \frac{\pi}{3} + k \times 2\pi = \frac{2\pi}{3} + k \times 2\pi \end{cases}, \quad k \in \mathbb{Z}$$

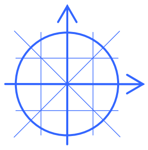
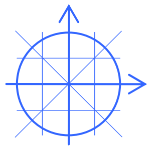
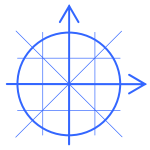
$$S_{\mathbb{R}} = \left\{ \frac{\pi}{3} + k \times 2\pi \ ; \ \frac{2\pi}{3} + k \times 2\pi \quad , \ k \in \mathbb{Z} \right\}$$



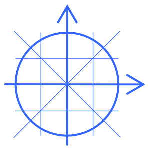
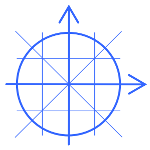
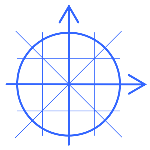
030 Résoudre dans :

1) $\mathbb{R}$ 2) $[\ 0\ ;\ 2\pi\ [$ 3) $]\ -\pi\ ;\ \pi\ ]$ et 4) $\left[\ 0\ ;\ \frac{\pi}{2}\ \right]$

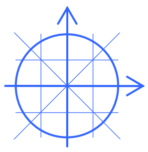
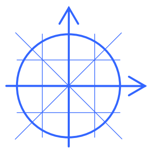
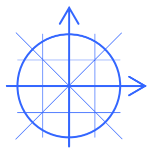
$\sin x = \frac{1}{2}$

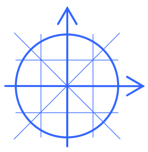
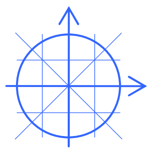
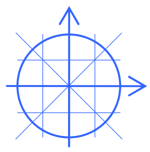
$\cos x = \frac{-1}{2}$

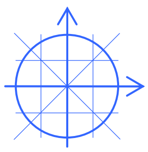
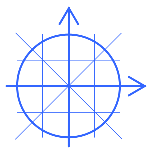
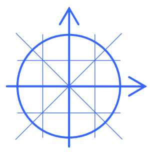
$\sqrt{2} \times \sin x - 1 = 0$

$3 \times \cos x - 5 = 0$

$\sin^2 x + 3 \times \sin x = 0$

L'amitié sans confiance, c'est une fleur sans parfum. Laure Conan