NOMBRES COMPLEXES

Équations Degré 1
Résolution dans $\mathbb{C}$

1) $3z - 4 = z + 2i$

$3z - z = 4 + 2i$

$2z = 4 + 2i$

$z = \frac{4 + 2i}{2}$

$\boxed{z = 2 + i}$

2) $-3(5 - 2z) = \frac{i}{2}$

$-15 + 6z = \frac{i}{2}$

$6z = 15 + \frac{i}{2}$

$z = \frac{15 + \frac{i}{2}}{6}$

$z = \left(15 + \frac{i}{2}\right) \times \frac{1}{6}$

$z = \frac{15}{6} + \frac{i}{12}$

$z = \frac{\cancel{3} \times 5}{\cancel{3} \times 2} + \frac{i}{12}$

$\boxed{z = \frac{5}{2} + \frac{i}{12}}$

3) $(3-2i)z = -3$

$$z = \frac{-3}{(3-2i)} \times \frac{(3+2i)}{(3+2i)}$$

quantité conjuguée $= 1$ pour supprimer les i au dénominateur

$$z = \frac{-3(3+2i)}{(3-2i)(3+2i)}$$

$\longrightarrow z \times \bar{z} = x^2 + y^2$

$$z = \frac{-9-6i}{3^2+2^2}$$

$$\boxed{z = -\frac{9}{13} - \frac{6i}{13}}$$

4) $i \times \bar{z} = i + 5$

$$\bar{z} = \frac{(i+5)}{i} \times \frac{i}{i}$$

$$\bar{z} = \frac{i^2 + 5i}{i^2}$$

$$\bar{z} = \frac{-1+5i}{-1}$$

$$\bar{z} = 1 - 5i$$

$$\boxed{z = 1 + 5i}$$

En effet :

si $z = x + iy$

alors $\bar{z} = x - iy$

5) $iz + 1 = -2z + i$

$2z + iz = -1 + i$

$z(2+i) = -1+i$

$$z = \frac{(-1+i)}{(2+i)} \times \frac{(2-i)}{(2-i)}$$

$$z = \frac{-2+i+2i-i^2}{2^2+1^2}$$

$$\boxed{z = \frac{-1}{5} + \frac{3i}{5}}$$

6) $$-5z = \frac{1-i}{i+3}$$

$$-5z = \frac{(1-i)}{(3+i)} \times \frac{(3-i)}{(3-i)}$$

$$-5z = \frac{3-i-3i+i^2}{3^2+1^2}$$

$$-5z = \frac{2-4i}{10} = \frac{1-2i}{5}$$

$$z = \frac{\frac{1-2i}{5}}{-5} = \frac{(1-2i)}{5} \times \frac{1}{-5}$$

$$z = \frac{1-2i}{-25}$$

$$\boxed{z = -\frac{1}{25} + \frac{2i}{25}}$$

7) $(3-i)\bar{z} = \dfrac{1+i}{1-i}$

$(3-i)\bar{z} = \dfrac{(1+i)}{(1-i)} \times \dfrac{(1+i)}{(1+i)}$

$(3-i)\bar{z} = \dfrac{(1+i)^2}{1^2+1^2} = \dfrac{\cancel{1}+\cancel{2}i+\cancel{i^2}}{\cancel{2}} = i$

$\bar{z} = \dfrac{i}{(3-i)} \times \dfrac{(3+i)}{(3+i)}$

$\bar{z} = \dfrac{3i+i^2}{3^2+1^2} = \dfrac{-1}{10} + \dfrac{3i}{10}$

$\boxed{z = -\dfrac{1}{10} - \dfrac{3i}{10}}$

8) $\dfrac{z+i}{\bar{z}-i} = 2 \quad , z \neq -i$

$z+i = 2(\bar{z}-i) = 2\bar{z} - 2i$

$z - 2\bar{z} = -2i - i = -3i$

or $z = x+iy$
et $\bar{z} = x-iy$

$x+iy-2(x-iy) = 0-3i$

$x+iy-2x+i2y = 0-3i$

$-x+i3y = 0-3i$

<u>Par identification</u> :

$-x=0 \qquad 3y=-3$

$x=0 \qquad y=-1 \qquad$ Donc $z=-i$

Finalement $\boxed{S = \emptyset}$ car $-i$ est une valeur interdite.

9) $3z + 5 - i = (1+i)\bar{z} - 1$

$3z - (1+i)\bar{z} = -5 - 1 + i = -6 + i$

$3(x+iy) - (1+i)(x-iy) = -6+i$

$3x + i3y - (x - iy + ix - i^2y) = -6+i$

$3x + i3y - (x - iy + ix + y) = -6+i$

$3x + i3y - x + iy - ix - y = -6+i$

$(2x - y) + i(4y - x) = -6+i$

Par identification :

$$\times 2 \begin{cases} 2x - y = -6 \\ -x + 4y = 1 \end{cases} \Leftrightarrow \begin{cases} 2x - y = -6 \\ -2x + 8y = 2 \end{cases}$$

par addition : $7y = -4$

$y = \frac{-4}{7}$

Et $2x - y = -6$

$2x + \frac{4}{7} = -6$

$2x = -6 - \frac{4}{7} = \frac{-46}{7}$

$x = \frac{-46}{7} \times \frac{1}{2} = \frac{-23 \times \cancel{2}}{7 \times \cancel{2}}$

$x = \frac{-23}{7}$

Finalement $\boxed{z = -\frac{23}{7} - \frac{4}{7}i}$

10) $(4i+1)\bar{z} + i = (2+5i)(z-i)$

$$(4i+1)\bar{z} + i = 2z - 2i + 5iz - 5i^2$$

$$(4i+1)\bar{z} - 2z - 5iz = -i - 2i + 5$$

$$(4i+1)\bar{z} - z(2+5i) = 5-3i$$

$$(4i+1)(x-iy) - (x+iy)(2+5i) = 5-3i$$

$$i4x - 4i^2y + x - iy - (2x + i5x + i2y + 5i^2y) = 5-3i$$

$$i4x + 4y + x - iy - (2x + i5x + i2y - 5y) = 5-3i$$

$$i4x + 4y + x - iy - 2x - i5x - i2y + 5y = 5-3i$$

$$(9y - x) + i(-x - 3y) = 5-3i$$

Par identification :

$$\begin{cases} -x + 9y = 5 \\ -x - 3y = -3 \end{cases}$$

Par soustraction

$$12y = 8$$
$$y = \frac{8}{12}$$
$$y = \frac{2}{3}$$

et $-x - \cancel{3} \times \frac{2}{\cancel{3}} = -3$

$$x + 2 = 3$$
$$x = 5$$

Finalement $\boxed{z = 5 + \frac{2}{3} i}$